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THE EFFECT OF WALL SUCTION AND THERMOPHORESIS ON AEROSOL PARTICLE DEPOSITION FROM A LAMINAR BOUNDARY LAYER ON A FLAT PLATE

A. F. Mills, Xu Hang and F. Ayazi
University of California, Los Angeles, CA 90024, U.S.A.

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NOMENCLATURE

$C_m, C_s, C_t, C_1, C_2, C_3$	constants in equation (2)
d	diameter
D	diffusion coefficient
f	dimensionless stream function, $\psi/(2u_e \nu x)^{1/2}$
g	dimensionless temperature, $(T-T_e)/(T_s-T_e)$
h_c	convective heat transfer coefficient
J	particle flux
k	thermal conductivity
Kn	Knudsen number
N	particle concentration
Pr	Prandtl number
Re	Reynolds number, $u_e x/\nu$
Sc	Schmidt number
St	Stanton number
T	temperature
u, v	streamwise and normal velocity components, respectively
V_T	thermophoretic velocity
x, y	streamwise and normal coordinates, respectively.

Greek symbols

δ_h	equivalent stagnant film thickness for heat transfer
ε	expansion parameter, $1/Sc_p$
ϕ	dimensionless particle concentration, N/N_e
η	similarity variable, $y(u_e/2\nu x)^{1/2}$
κ	thermophoresis parameter, $-V_T(T_s-T_e)/\nu(\partial T/\partial y)$
ν	kinematic viscosity
ψ	stream function.

Subscripts

e	free stream
h	heat transfer
o	outer solution
p	particle
s	surface.

Superscripts

C	convection
D	Brownian diffusion
T	thermophoresis
′	differentiation with respect to η
*	limit of zero mass transfer rate.

1. INTRODUCTION

There are numerous technological and environmental problems which involve deposition of aerosol particles from a condensing vapor–gas mixture. Of current concern is deposition of radioactive aerosol particles inside a nuclear reactor containment following a hypothetical accident, which plays a critical role in reducing the amount of radioactivity that might be predicted to subsequently escape to the atmosphere. Deposition may occur on the walls of the pressure vessel, in pipes, or in bubbles rising in the pressure suppression pool. Mechanisms of deposition include Brownian diffusion, thermophoresis, diffusiophoresis, convection, inertial and sedimentation. Current engineering practice, e.g. computer codes such as HAARM-3 [1], PARDESIKO [2] and SUPRA [3], etc. is to calculate the deposition rates separately, and then to simply add them to obtain the total deposition rate. The purpose of this note is to examine the validity of such engineering calculation procedures.

The major situation of interest for nuclear reaction safety is deposition of particles from a steam-condensable gas mixture, where the gas might be air, hydrogen or a mixture of both. The flow may be forced or buoyancy driven; past a vertical or horizontal surface, laminar or turbulent, or could be a vortex flow inside a bubble. For all particle sizes the convective deposition associated with steam condensation plays an important role. For small particles, $d_p \lesssim 0.1$ μm, the diffusive mechanisms, i.e. Brownian diffusion, diffusiophoresis and thermophoresis can be important. For large particles, $d_p \gtrsim 1.0$ μm, inertial deposition and sedimentation can be important. In this note we consider only the small particles and study the coupling of convective and diffusive mechanisms of particle deposition.

The analysis of coupled deposition mechanisms has received some attention in the aerosol science and technology literature. Most of the published work deals with the coupling of Brownian diffusion and sedimentation, which is of limited interest in the current context. There has been some work on coupled thermophoresis and Brownian diffusion [4–6]. The most pertinent study is that of Hales *et al.* [7], who analyzed coupled convective and diffusive deposition of small aerosol particles in the laminar boundary layer associated with film condensation of steam from a stationary steam–air mixture on a vertical wall. The results, although correct and useful, do not clearly display the interactions between the various deposition mechanisms. We therefore chose to solve a simpler model problem in which transport and thermodynamic properties are taken to be constant. But in order to assume constant properties it is also necessary to ignore diffusiophoresis, since this phenomenon is related to the difference between the mass average and molar average velocities, which can only be calculated in a variable property analysis. The results of the analysis do, however, clearly display the interactions of the convective, Brownian diffusive and thermophoretic mechanisms of deposition, and allow an evaluation of approximations currently used in engineering practice.

2. ANALYSIS

The flow considered is the laminar forced convection air boundary layer on a flat plate. The plate is imagined to be porous and there may be suction or blowing through the plate. The plate is maintained at a temperature T_s, different to the free stream air temperature T_e. Constant properties are assumed. The particles are small enough for inertial effects to be ignored, and their concentration is too low for coagulation to occur at an appreciable rate. The governing partial differential equations admit a similarity solution and methods for obtaining the dimensionless stream function $f(\eta)$ and dimensionless temperature $g(\eta)$ are straightforward and well known. The particle conservation equation is less familiar, and may be written as

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=D\frac{\partial^2 N}{\partial y^2}+\frac{\partial}{\partial y}(V_T N), \tag{1}$$

where the preferred expression for the thermophoretic velocity V_T is that given by Talbot *et al.* [8]

$$V_T=\frac{2C_s\nu\left(\frac{k_{tr}}{k_p}+2C_t Kn\right)[1+Kn(C_1+C_2\,e^{-C_3/Kn})]\left(\frac{1}{T}\frac{\partial T}{\partial y}\right)}{(1+6C_m Kn)\left(1+2\frac{k_{tr}}{k_p}+4C_t Kn\right)}, \tag{2}$$

and the boundary conditions are

$$\begin{aligned} &y=0: N=0\ \text{(totally absorbing wall)};\\ &y\to\infty, x\leqslant 0: N=N_e. \end{aligned} \tag{3}$$

The self-similar form of equation (1) in terms of the dimensionless particle concentration ϕ is

$$\phi''+Sc_p(f-\kappa g')\phi'-(Sc_p\,\kappa g'')\phi=0, \tag{4}$$

with boundary conditions

$$\eta=0,\quad \phi=0;\quad \eta\to 1,\quad \phi=1, \tag{5}$$

and

$$\kappa=\frac{2C_s\left(\frac{k_{tr}}{k_p}+2C_t Kn\right)[1+Kn(C_1+C_2\,e^{-C_3/Kn})](T_s-T_e)}{T(1+6C_m Kn)\left(1+2\frac{k_{tr}}{k_p}+4C_t Kn\right)}. \tag{6}$$

The particle size range of interest is spanned by a range of particle Schmidt numbers from 10^3 to 10^6. For $T = 300$ K and $T_s-T_e = 3$, 30, and 300 K, the corresponding values of the thermophoresis parameter κ are approximately 0.01, 0.1, and 1, respectively.

3. SOLUTIONS

Solutions for f and g for various values of the Prandtl number and the blowing parameter $f(0) = -2^{1/2}\,Re_x^{1/2}(v_s/u_e)$ are of course available: however, in order to solve equation (4) detailed profiles are required, so solutions were obtained by formal integration and iteration for a Prandtl number of 0.7 and a range of values for $f(0)$. To solve the particle equation, equation (4), it was first divided by Sc_p and with $\varepsilon = 1/Sc_p$ to obtain

$$\varepsilon\phi''+(f-\kappa g')\phi'+(-\kappa g'')\phi=0. \tag{7}$$

Defining

$$a(\eta)=f(\eta)-\kappa g'(\eta);\quad b(\eta)=-\kappa g''(\eta), \tag{8a,b}$$

there is obtained

$$\varepsilon\phi''+a(\eta)\phi'+b(\eta)\phi=0. \tag{9}$$

Direct solution of equation (9) is made difficult by the fact that ε is very small (10^{-3}–10^{-6}). However, for the present purpose it is sufficient to obtain the outer solution, for which

$$a\phi'+b\phi=0;\quad \eta\to\infty,\quad \phi=1. \tag{10}$$

Integration gives

$$\phi_o(\eta)=\exp\left[\int_\eta^\infty \frac{b(\eta)}{a(\eta)}\,d\eta\right]. \tag{11}$$

The situation of primary interest is a cold wall ($T_s < T_e$) with suction, which can be considered to be a model problem for deposition of aerosol particles from a condensing superheated vapor. Since $\kappa > 0$, $g'(\eta)\leqslant 0$ and $f(\eta)\geqslant 0$, it follows that $g''(\eta)\geqslant 0$ and $b(\eta)/a(\eta)\leqslant 0$. Thus $\phi_o(\eta)$ decreases monotonically as the wall is approached, and has a finite value at $\eta = 0$. The deposition rate is obtained from the outer solution as follows

$$\begin{aligned} J_s &= J_s^C+J_s^T,\\ &= \lim_{y\to 0}\,[N(v+V_T)],\\ &= \lim_{y\to 0}\,[N_e\phi_o(v+V_T)], \end{aligned} \tag{12}$$

or

$$J_s=N_e\phi_o(0)\,[v_s+V_{T,s}]. \tag{13}$$

But

$$v_s=-\left(\frac{u_e\nu}{2x}\right)^{1/2}f(0);\quad V_{T,s}=\nu\kappa\frac{\partial T/\partial y|_{y=0}}{(T_s-T_e)}=\kappa g'(0)\left(\frac{u_e\nu}{2x}\right)^{1/2},$$

thus

$$J_s=N_e\phi_o(0)\left(\frac{u_e\nu}{2x}\right)^{1/2}[-f(0)+\kappa g'(0)]. \tag{14}$$

If a Stanton number for particle deposition is defined as $St_p = -J_s/N_e u_e$, then

$$St_p\,Re_x=2^{-1/2}\phi_o(0)[f(0)-\kappa g'(0)]. \tag{15}$$

As mentioned in the Introduction, current engineering practice in nuclear safety work is to assume that the various mechanisms of aerosol deposition are additive. Thus for this model problem

$$J_s=J_s^D+J_s^T+J_s^C, \tag{16}$$

where

$$J_s^D\simeq -u_e N_e\,St_p^D \tag{17a}$$

$$J_s^T\simeq N_e V_{T,s}=-u_e N_e\kappa\,Pr\,St_h\ \text{(stagnant film model)}, \tag{17b}$$

$$J_s^C\simeq N_e v_s=-N_e(u_e\nu/2x)^{1/2}f(0). \tag{17c}$$

Hence

$$St_p=[St_p^D+\kappa\,Pr\,St_h+2^{-1/2}Re_x^{-1/2}f(0)]. \tag{18}$$

Also

$$\frac{St_h}{St_h^*}=\frac{g'(0)}{g'(0)^*};\quad \frac{St_p^D}{St_p^{D*}}\simeq\frac{g'(0)}{g'(0)^*}, \tag{19a,b}$$

where the asterisk denotes values in the limit of zero suction, and appropriate correlations are

$$St_h^*=0.332Re_x^{-1/2}\,Pr^{-2/3};\quad St_p^{D*}=0.332Re_x^{-1/2}\,Sc_p^{-2/3}.$$

Substitution in equation (18) gives

$$St_p\,Re_x^{1/2}=0.707f(0)+0.332[Sc_p^{-2/3}+\kappa\,Pr^{1/3}][g'(0)/g'(0)^*], \tag{20}$$

where for $Pr = 0.7$, $g'(0)^* = -0.4139$, and $Pr^{1/3} = 0.888$.

Table 1 shows a comparison of equations (15) and (20) for $Pr = 0.7$, $Sc_p = 10^3$–10^6; $\kappa = 0.01$, 0.1, and 1, and $f(0) = 0$–5.

Table 1. Comparison of deposition rates given by equations (15) and (20) for suction on a cold wall (simulating condensation from a vapor–noncondensable gas mixture)

	$St_p Re_x^{1/2}$														
		$\kappa = 0.01$					$\kappa = 0.1$					$\kappa = 1.0$			
		Equation (20)					Equation (20)					Equation (20)			
	Equation	Sc_p				Equation	Sc_p				Equation	Sc_p			
$f(0)$	(15)	10^6	10^5	10^4	10^3	(15)	10^6	10^5	10^4	10^3	(15)	10^6	10^5	10^4	10^3
0.0	0.00291	0.00298	0.00310	0.00366	0.00628	0.02773	0.02951	0.02963	0.03019	0.03280	0.2095	0.2948	0.2949	0.2955	0.2981
0.5	0.3959	0.3584	0.3587	0.3596	0.3639	0.3767	0.4026	0.4028	0.4037	0.4081	0.5346	0.8443	0.8445	0.8455	0.8498
1.0	0.7091	0.7141	0.7144	0.7157	0.7219	0.7265	0.7771	0.7773	0.7787	0.7849	0.08619	1.4071	1.4074	1.4087	1.4149
2.0	1.4157	1.4256	1.4260	1.4282	1.4383	1.4284	1.5286	1.5290	1.5312	1.5413	1.5302	2.5586	2.5590	2.5612	2.5713
3.0	2.1224	2.1373	2.1379	2.1410	2.1552	2.1323	2.2821	2.2828	2.2858	2.3001	2.2119	3.7305	3.7312	3.7342	3.7485
4.0	2.8293	2.8491	2.8499	2.8539	2.8723	2.8373	3.0368	3.0376	3.0416	3.0600	2.9017	4.9138	4.9146	4.9186	4.9370
5.0	3.5363	3.5610	3.5620	3.5669	3.5896	3.5429	3.7921	3.7932	3.7981	3.8207	3.5966	6.1035	6.1046	6.1095	6.1321

Table 2. Comparison of deposition rates given by equations (15) and (20) for blowing from a cold wall (simulating evaporation into a vapor–noncondensable gas mixture)

	$St_p Re_x^{1/2}$														
		$\kappa = 0.01$					$\kappa = 0.1$					$\kappa = 1.0$			
		Equation (20)					Equation (20)					Equation (20)			
	Equation	Sc_p				Equation	Sc_p				Equation	Sc_p			
$f(0)$	(15)	10^6	10^5	10^4	10^3	(15)	10^6	10^5	10^4	10^3	(15)	10^6	10^5	10^4	10^3
0.000	0.0029	0.0030	0.0031	0.0037	0.0063	0.0277	0.0295	0.0296	0.0302	0.0328	0.2095	0.2948	0.2949	0.2955	0.2981
−0.001	0.0022	0.0023	0.0024	0.0030	0.0056	0.0270	0.0288	0.0289	0.0295	0.0321	0.2088	0.2938	0.2939	0.2945	0.2971
−0.004	0.0001	0.0001	0.0003	0.0008	0.0034	0.0249	0.0265	0.0267	0.0272	0.0298	0.2068	0.2905	0.2906	0.2912	0.2938
−0.005	—	—	—	0.0001	0.0027	0.0242	0.0258	0.0259	0.0265	0.0291	0.2062	0.2894	0.2896	0.2901	0.2927
−0.010	—	—	—	—	—	0.0207	0.0221	0.0222	0.0227	0.0253	0.2029	0.2840	0.2842	0.2847	0.2873
−0.040	—	—	—	—	—	—	—	—	0.0004	0.0029	0.1830	0.2516	0.2517	0.2523	0.2547
−0.050	—	—	—	—	—	—	—	—	—	—	0.1764	0.2408	0.2410	0.2415	0.2439
−0.100	—	—	—	—	—	—	—	—	—	—	0.1429	0.1870	0.1871	0.1876	0.1899
	—	—	—	—	—	—	—	—	—	—	0.0734	0.0710	0.0801	0.0805	0.0825
−0.250	—			—		—	—	—	—	—	0.0344	0.0268	0.0269	0.0273	0.0291
−0.275	—	—	—	—	—	—	—	—	—	—	—	0.0003	0.0004	0.0007	0.0025
−0.277	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.0003
−0.278	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

Table 2 shows a similar comparison for blowing with $f(0) = 0$ to -0.278.

4. DISCUSSION

Equation (15) can be shown to be the first term in the series obtained if an exact solution to equation (7) is sought using singular perturbation methods (for example, as was done by Walker *et al.* [9]). It is accurate for $Sc_p \gtrsim 10^3$, and thus can be regarded as an exact solution for the present purpose. Equation (20) is an approximate result obtained by simply adding the deposition rates for each mechanism if they are assumed to act independently, and also if it is assumed that thermophoretic deposition can be calculated using the temperature gradient of a stagnant film model, i.e. $\partial T/\partial y$ in equation (2) is approximated as $(T_s - T_e)/\delta_h$, $\delta_h = k/h_c$, to obtain equation (17b). Tables 1 and 2 show that the agreement is good only when convection dominates, e.g. $f(0) = 5.0$, $\kappa = 0.01$ in Table 1. However, the tables also show that when thermophoresis dominates, e.g. $f(0) = 0$, $\kappa = 1.0$ in Table 1, the agreement is poor. This discrepancy is due to the failure of the approximate method to recognize that Brownian diffusion does play an important role in the inner region of the concentration boundary layer, even for large Sc_p and κ: since $N \to 0$ at the wall, Brownian diffusion dominates as $y \to 0$. The approximate method in using the stagnant film model to give an average temperature gradient for the calculation of the thermophoretic velocity [equation (17b)], overestimates the rate of deposition by nearly 50% for $\kappa = 1.0$.

For the condensation of superheated steam–noncondensable gas mixtures, both κ and $f(0)$ can be large, and Table 1 shows that the approximate equation can overestimate deposition rates by almost 100%. A situation of particular importance to boiling water nuclear reactor safety is the scrubbing of aerosol particles from nearly pure gas bubbles rising through a saturated suppression pool: in such a case there is evaporation into the bubbles and the only removal mechanism is thermophoresis: since the removal rates are then very low the situation is particularly dangerous. Table 2 shows that for low evaporation rates, $0 < -f(0) < 0.01$, the approximate equation overestimates the removal rate by nearly 50%.

Equation (19a) is exact and allows an unambiguous evaluation of the coupling between thermophoresis and convection. However, choice of an appropriate 'blowing factor' for the particle Stanton number, and hence the equivalent stagnant film thickness for Brownian diffusion, is not straightforward. Possibilities which have, or may be considered, for use in industry include (1) unity, e.g. refs. [1, 3], (2) the factor appropriate for the limit $Sc_p \to \infty$, as given by Bird *et al.* [10], and (3) the same factor as that used for heat transfer, i.e. equation (19b), as used herein. Our choice was somewhat arbitrary, but the important conclusions of our study are not altered if an alternative choice is made.

In conclusion we suggest that the results displayed here indicate that current engineering methods of calculating deposition rates of small aerosol particles can be significantly in error, and that proper accounting for the coupling between the various deposition mechanisms is required to obtain improved results.

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HEAT TRANSFER BETWEEN HOT COMBUSTION GASES AND A COLD WALL IN NARROW CHANNELS FOR LIMIT FLAMES

JÓZEF JAROSIŃSKI
Institute of Aeronautics, 02-256 Warsaw, Al. Krakowska 110/114, Poland

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NOMENCLATURE

c_p	specific heat at constant pressure
d	the 4/5 length of the high temperature zone
D	width of the narrow channel, equal to the quenching distance for limit flames
Nu	Nusselt number as related to the quenching distance D, $\alpha D/\lambda_b$
Pe	Péclet number expressed in terms of the parameters of the hot combustion gases for limit flames, $c_{pb}\rho_b u_b D/\lambda_b$
Pe_L	Péclet number expressed in terms of the parameters of the fresh mixture for limit flames, $c_{pu}\rho_u u_L D/\lambda_u$
St	Stanton number, Nu/Pe
T	absolute temperature
u	flow speed
u_L	burning velocity
x	dimensionless coordinate normal to the flame front
z	distance normal to the flame front.